Relationship Between Power Sequences And Their (N-1)th Difference Sequence

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Abstract— This paper seeks to find a relation between terms of power sequences such as square and cubic sequences and their (n-1)th difference sequences

Index Terms— Arithmetic Progression, Power Sequences, Difference Sequences, Factorial

1 INTRODUCTION

IT is said that there is no relationship between squares of natural numbers. But that is not so. There is definite pattern their difference is an arithmetic progression. Similar thing happens with cubic sequence. Its second difference is in arithmetc progression. And these arithmetic progression's common difference is the factorial of the power. This paper seeks to elucidate this pattern and reach a definite theorem

2 TERMS USED IN PAPER

2.1 Power Sequences with base N

Sequence of all whole numbers raised to power N
Example-
Power sequence with base 2

0	1	4	9	16	25	36	49
	64	81	100				

Power sequence with base 3

0	1	8	27	64	125	216
	343	512	729	1000		

2.2 Nth Difference Sequences

0th difference sequence-The sequence itself
1st difference sequence-Sequence of difference between the successive terms of power sequence
2nd difference sequence- Sequence of difference between successive terms of 1st difference sequence
3rd difference sequence- Sequence of difference between successive terms of 2nd difference sequence

3 HYPOTHESES

(N-1)th difference sequence of Power sequence with base N would be an arithmetic progression with common difference as N!

4 PROOFS

4.1 PROOF OF THEOREM FOR N=1

Hypothesis based on equation
As N=1, so
(1-1)th difference sequence would be an arithmetic progression with common difference=1!
=>0th difference sequence would be an arithmetic progression with common difference=1

But as 0th makes no sense so it is 1st difference sequence. Its similar to as 0! = 1!
Proof
Power sequence with base 1

0	1	2	3	4	5	6	7
	8	9	10				

0th difference sequence

0	1	2	3	4	5	6	7
	8	9	10				

Common difference =1

As proof justifies hypothesis thus the theorem is proven

4.2 PROOF OF THEOREM FOR N=2

Hypothesis based on equation
For N=2, so Hypothesis based on equation
At N=2, so
(2-1)th difference sequence would be an arithmetic progression with common difference=2!
=>1st difference sequence would be an arithmetic progression with common difference=2

Proof
Power sequence with base 2

0	1	4	9	16	25	36	49
	64	81	100				

1st difference sequence

1	3	5	7	9	11	13	15
	17	19					

Common difference =2

As proof justifies hypothesis thus the theorem is proven

4.3 PROOF OF THEOREM FOR N=3

Hypothesis based on equation
For N=3, so Hypothesis based on equation
At N=3 so
(3-1)th difference sequence would be an arithmetic progression with common difference=3!
=>2nd difference sequence would be an arithmetic progression with common difference=6

Proof

Power sequence with base 3

0 1 8 27 64 125 216
343 512 729 1000

1st difference sequence

1 7 19 37 61 91 127
169 217 271

2nd difference sequence

6 12 18 24 30 36 42 48
54

Common difference =6

As proof justifies hypothesis thus the theorem is proven

4.4 PROOF OF THEOREM FOR N=4

Hypothesis based on equation

For N=4, so Hypothesis based on equation

At N=4, so

(4-1)th difference sequence would be an arithmetic progression with common difference=4!

=>3rd difference sequence would be an arithmetic progression with common difference=24

Proof

Power sequence with base 4

0 1 16 81 256 625 1296
2401 4096 6561 10000

1st difference sequence

1 15 65 175 369 671 1105
1695 2465 3439

2nd difference sequence

14 50 110 194 302 434 590
770 974

3rd difference sequence

36 60 84 108 132 156 180
204

Common difference =6

As proof justifies hypothesis thus the theorem is proven

4.5 PROOF OF THEOREM FOR N=5

Hypothesis based on equation

For N=5, so Hypothesis based on equation

At N=5, so

(5-1)th difference sequence would be an arithmetic progression with common difference=5!

=>4th difference sequence would be an arithmetic progression with common difference=120

Proof

Power sequence with base 5:

0 1 128 243 1024 3125 7776
16807 32768 59049 100000

1st difference sequence

1 31 211 781 2101 4651 9031
15961 26281 40951

2nd difference sequence

30 180 570 1320 2550 4380 6930
10320 14670

3rd difference sequence: 150,390,750,1230,1830,2550,3390,4350

4th difference sequence: 240,360,480,600,720,840,960

Common difference =120

As proof justifies hypothesis thus the theorem is proven

4.6 PROOF OF THEOREM FOR N=6

Hypothesis based on equation

For N=6, so Hypothesis based on equation

At N=6, so

(6-1)th difference sequence would be an arithmetic progression with common difference=6!

=>5th difference sequence would be an arithmetic progression with common difference=720

Proof

Power sequence with base 6

0 1 64 729 4096 15625
46656 117649 262144 531441 1000000

1st difference sequence:

1 63 665 3367 11529 31031 70993
144495 269297 468559

2nd difference sequence:

62 602 2702 8162 19502 39962 73502
124802 199262

3rd difference sequence:

540 2100 5460 11340 20460 33540 51300
74460

4th difference sequence:

1560 3360 5880 9120 13080 17760 23160

5th difference sequence:

1800 2520 3240 3960 4680

Common difference =720

As proof justifies hypothesis thus the theorem is proven

4.7 PROOF OF THEOREM FOR N=7

Hypothesis based on equation

For N=7, so Hypothesis based on equation

At N=7, so

(7-1)th difference sequence would be an arithmetic progression with common difference=1!

=>6th difference sequence would be an arithmetic progression with common difference=5040

Proof

Power sequence with base 7

0 1 128 2187 16384 78125
279936 823543 2097152 4782969 10000000

1st difference sequence:

1 127 2059 14197 61741 201811 543607
1273609 2685817 5217031

2nd difference sequence:

126 1932 12138 47544 140070 341796 730002
1412208

3rd difference sequence:

1806 10206 35406 92526 201726 388206 682206
1119006

4th difference sequence:

8400 25200 57120 109200 186480 294000 436800

5th difference sequence:

16800 31920 52080 77280 107520 142800

6th difference sequence:

15120 20160 25200 30240 35280

Common difference =5040

As proof justifies hypothesis thus the theorem is proven

5 Conclusion

As theorem can be proven for all natural numbers similar to above proofs. Hence, it's a valid theorem. I propose to call it Saxena Theorem